Adaptive Refinement for Neutron Transport

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1 Introduction

The goal of my project was to apply adaptive spatial refinement to the neutron transport equation in two spatial dimensions. Under certain assumptions neutron transport is described by the following time independent one-group linear Boltzmann equation:

$$(\Omega \cdot \nabla + \sigma_t(I - P) + \sigma_a P)\Psi = Q$$

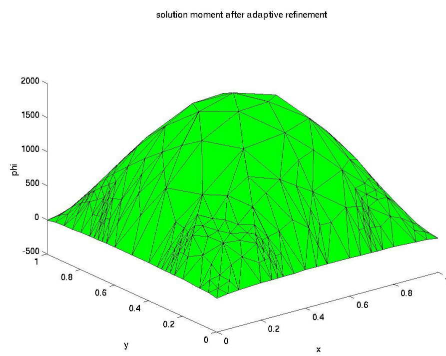
The P operator represents integration over all angles, and the solution is the angular flux, a function of both space and angle. While several methods exist for tackling this equation, I focused on spherical harmonics with finite elements, the FE-PN method. The discretization was accomplished through a least squares approach, where the boundary conditions are enforced by means of a boundary functional which is incorporated into the least squares functional. This concept is taken from the paper by Manteuffel [2]. One major advantage of the least squares approach is that it provides an error estimate, since the value of the functional that is being minimized can be calculated. This calculation is done element-wise, and therefore serves as the criterion for the adaptive refinement. The code was written in Matlab and has the ability to handle the following:

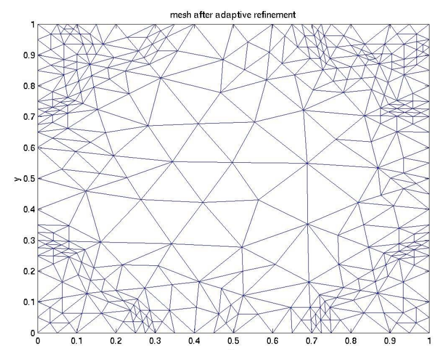
- vacuum (zero or non-zero inflow) and reflective boundary conditions
- different values of absorption and scattering cross section in different regions

- any order of spherical harmonic expansion
- adaptive refinement in space based on the least squares functional

2 Results

After running the code on quite a few test problems, it is clear that the adaptive refinement scheme is quite effective in the diffusive regime, where the mean free path for scattering is small compared to the diameter of the domain. In the thin regime, where there are relatively few scattering events, spatial refinement has little effect. The accuracy is these cases can instead be improved by using a higher order expansion in spherical harmonics. This result seems reasonable since in the thin regime the particle activity and therefore the solution is dominated by streaming, which has a high degree of angular dependency. On the other hand, in the diffusive regime scattering is so frequent that most angular dependency would vanish. This concept is further supported by the discretization bounds given in Manteuffel [2], which have both an angular and a spatial component. The angular component has in it's denominator the order of the angular expansion, while the spatial component has in it's numerator a factor relating tothe mesh size. In the thin regime, the angular component dominates, and increasing order is effective while decreasing mesh size is not. In the diffusive regime, the opposite is true, and spatial refinement is effective. The following table gives a numerical example in the two regimes. For this example, the domain is the unit square, and source term q has a value of 100 everywhere, and the boundary conditions are zero inflow (vacuum). For the thin regime the absorption cross section is 1 and the scattering cross section is 0, while these values are .01 and 100 in the diffusive regime respectively. The figures below show the mesh and one of the solution moments for the diffusive example with 4 iterations of adaptive refinement.





Thin Regime	relative error	improvement
initial mesh, first order in angle	.0673	–
after 4 iterations of adaptive refinement	.0669	< 1 %
initial mesh, fifth orer in angle	.0082	87 %

Diffusive Regime	relative error	improvement
initial mesh, first order in angle	.0072	–
after 4 iterations of adaptive refinement	.0014	81 %
initial mesh, fifth orer in angle	.0072	0 %

References

[1] T.M. Austin, Advances on a Least-Squares Method for the 3-D Linear Boltzmann Equation, PhD thesis, University of Colorado at Boulder, 2001.

[2] T. A. Manteuffel, K. Ressel, G. Starke. A boundary functional for the least-squares finite element solution of neutron transport problems. SIAM J. Numer. Anal., 37:556-586, 2000.